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### Ohio State Engineer

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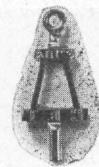
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# Some Engineering Applications of the Buckingham Pi Theorem



By EVERETT E. EDDEY

In attempting to solve a problem, an engineer often tries analytic mathematical methods first. Equations stating the relations that must be satisfied are set up. Solution of these equations then gives the desired relations existing between the different variables in the problem. Very often, however, it happens that it is very difficult or even impossible to set up and solve analytical equations. In such cases dimensional analysis will frequently give a solution. The Buckingham pi theorem is a very powerful tool of dimensional analysis. This theorem is particularly well adapted for use by engineers since little mathematical knowledge is necessary. The underlying physical principles, however, must be well known. This theorem was formulated by Dr. Edgar Buckingham of the Bureau of Standards.<sup>1</sup> The purpose of this article is to set forth this theorem and to show some applications of its uses.

As an introductory problem leading up to the theorem, consider the following very simple example: Suppose it is desired to find the time  $t$  required for an object traveling at a constant velocity  $v$  to traverse a distance  $d$ . The relation  $t=d/v$  is unknown, and it is desired to solve the

problem by dimensional analysis. A start is made by assuming (justification for this assumption will be given later) that the equation for  $t$  takes the following form:

$$t=Cd^aV^b$$

where  $C$  is a numerical constant and  $a$  and  $b$  are unknown exponents. Since  $t$  has the dimension of time;  $d$ , that of length; and  $v$ , that of length divided by time, it is obvious that if the equation is to be dimensionally consistent,  $a$  must equal 1 and  $b$  must be  $-1$ . Hence

$$t=C(d/v)$$

The constant  $C$  may be evaluated rather easily in the laboratory by a single experiment, for example, by pulling a cart at constant velocity and measuring the time required to travel a given distance.

Although the above example is rather trivial, it illustrates one important point: a constant in an equation was evaluated by a single experiment. The equation thus formed, however, applies in all cases. The Buckingham pi theorem is basically just a refinement of the method used in this example. The pi theorem is somewhat more sophisticated and may be applied to more intricate problems.

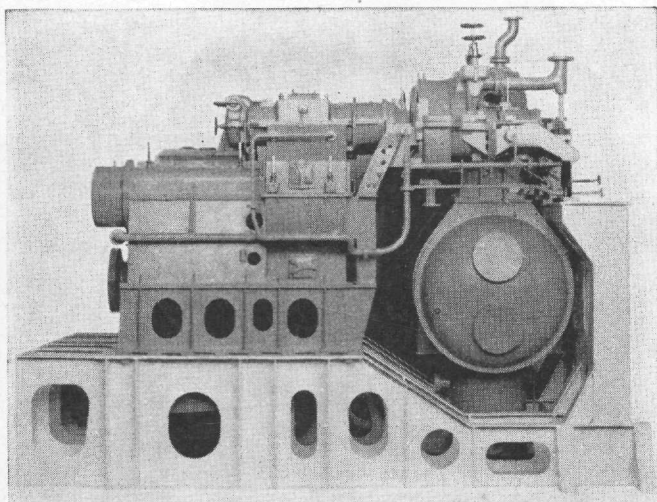
The Buckingham pi theorem states that if  $n$  quantities (force, viscosity, displacement, etc.) are concerned in a problem and if these  $n$  quantities are expressible in terms of  $m$  dimensions (mass, length, time, etc.), then there will be  $(n-m)$  independent dimensionless groups which can be formed by combinations of the quantities. If the  $(n-m)$  groups are called  $P_1, P_2, P_3$ , etc., then the solution of the problem must take the form:

$$F(P_1, P_2, P_3, \dots) = 0$$

It will be noted that this equation can be solved for any particular pi, say  $pi_1$ :

$$P_1=f(P_2, P_3, \dots)$$

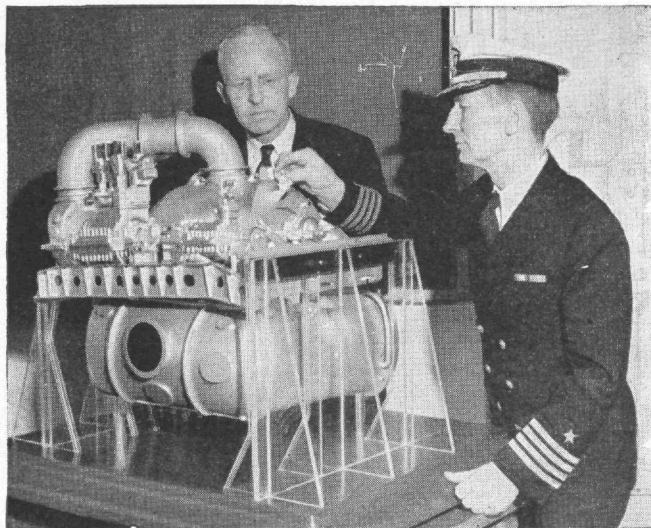
Several examples illustrating the use of the pi theorem will now be given. First, however, it might be well to explain the notation to be used.



—Courtesy General Electric.

Model of propulsion steam turbine gear unit with condenser for 6000-hp Victory ship

<sup>1</sup> Buckingham, E., *Phys. Rev.*, Vol. 4, p. 345, 1914.



—Courtesy General Electric.

**Capt. J. S. Evans, Inspector of naval material at Schemmady, examines a model of the new cross-compound propulsion turbine for destroyers**

The symbol for a dimension will be taken as the capital of the first letter in the name of the dimension. Thus M is taken as the symbol for mass; L, for length; T, for time, etc. Then dimensional formulas may be formed:

$$\text{velocity} = V = L/T$$

$$\text{density} = r = M/L^3$$

$$\text{force} = F = ML/T^2$$

As the first example, consider the problem of a naval architect who wishes to plot the water resistance to motion of a ship versus the velocity of the ship. An analytical approach was tried but difficulties were encountered so the pi theorem was resorted to. The force of water resistance will in general depend upon the shape of the ship. However if consideration is limited to ships of similar shape, then shape need no longer be considered. In this case some "characteristic linear dimension" should be specified to indicate relative size. Then the important quantities in connection with this problem are:

$$P = \text{force on the ship} = ML/T^2$$

$$S = \text{immersed surface area} = L^2$$

$$l = \text{a linear dimension (to indicate relative size)} = L$$

$$\nu \text{ (Nu)} = \text{kinematic viscosity of water} = L^2/T$$

$$V = \text{velocity of the ship} = L/T$$

$$r = \text{density of the fluid} = M/L^3$$

It will be seen that there are six quantities and three dimensions. Hence  $6 - 3 = 3$  pi's are ex-

pected. The general expression for a pi takes the shape:

$$\Pi = P^a S^b l^c \nu^d V^e r^f$$

Substituting the dimensional equivalents:

$$\Pi = (M^a L^a / T^{2a}) (L^{2b}) (L^c) (L^{2d} / T^d) (L^e / T^e) (M^f / L^{3f})$$

If the pi is to be dimensionless, the exponents of M, L, and T must add up to zero separately. Whence:

$$a + f = 0 \quad (\text{For } M)$$

$$a + 2b + c + 2d + e - 3f = 0 \quad (\text{For } L)$$

$$-2a - d - e = 0 \quad (\text{For } T)$$

There are six unknowns and three equations; hence, values may be arbitrarily assigned to three variables and the values of the other three variables determined from the above equations. Thus for  $\Pi_1$  choose  $a = 1, c = 0, e = 0$ . Inserting these values in the equations, one finds that  $f = -1, d = -2, b = 0$ .

$$\Pi_1 = \frac{P}{V^2 r}$$

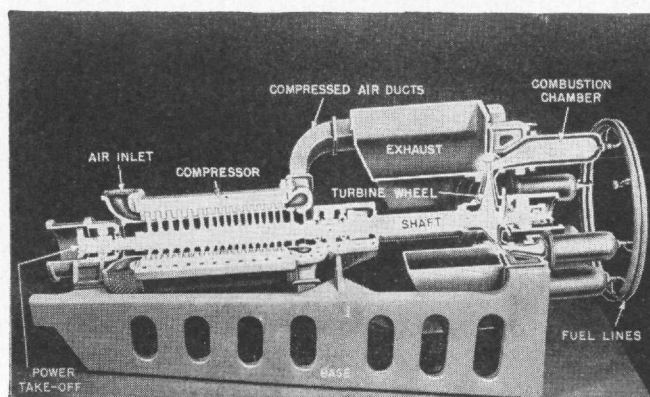
For  $\Pi_2$ , choose  $a = 0, b = 0, e = 1$ . Solving the system of equations, one discovers that  $f = 0, c = 1, d = -1$ .

$$\Pi_2 = \frac{lV}{\nu}$$

Since  $l^2/S$  is obviously a dimensionless quantity,

$$\Pi_3 = \frac{l^2}{S}$$

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—Courtesy General Electric.

**Model of a gas turbine for marine use**

The Ohio State Engineer

## BUCKINGHAM PI THEOREM

(Continued from page 8)

Then from the pi theorem:

$$F(P/v^2r, IV/v, l^2/S)=0$$

If attention is restricted at present to ships which are similarly loaded (that is, each ship displaces relatively the same amount of water), then the ratio  $l^2/S$  will be the same for all ships and may be omitted here. Then

$$F'(P/v^2r, IV/v)=0$$

$$\text{Solving for } P/v^2r=f(IV/v)$$

Now our naval architect could experiment on a model boat, varying the velocity of the model boat with respect to the fluid in which it floats and measuring the force upon the boat. Then if instead of merely plotting  $P$  versus  $V$ , he were to plot  $P/v^2r$  vertically against  $IV/v$  horizontally, he would have a curve which, according to the pi theorem, applies to all ships. To find the curve of  $P$  versus  $V$  for any ship, it is merely necessary to alter the co-ordinates of the above curve by the appropriate values of  $r$ ,  $v$  and  $l$ .

The simplicity resulting from the use of the pi theorem is apparent from these results. A single curve shows the effects of five variables. If the data were treated in the more conventional manner, plotting  $P$  versus  $V$  with  $l$ ,  $r$  and  $v$  as parameters, and if just five values of each parameter were considered, it would be necessary to plot  $5^3=125$  curves.

It might be that the naval architect would wish data on just a few specific points instead of a complete curve. In this case the analysis is somewhat simpler. Let primes refer to quantities connected with the model boat, and let quantities without primes refer to the full-sized ship. Then

$$P/v^2r=f(IV/v) \quad (1)$$

$$P'/v'^2r'=f(l'V'/v') \quad (2)$$

Now if the argument of the function  $f$  (that is,  $IV/v$ ) is the same for both the model and the full-sized ship, then  $f$  will have the same value in each case. Hence if

$$\begin{aligned} IV/v &= l'V'/v' \\ V' &= (v'l/vl')V \end{aligned} \quad (3)$$

then (1) divided by (2):

$$\begin{aligned} P &= (v^2r/v'^2r')P' \quad (4) \\ (P/n^2r) &= (P'/n'^2r') = f(IV/v)/f(l'V'/v') \end{aligned}$$

Hence if the model is given the velocity indicated by equation (3), then the force on the model and

the real ship will be related by (4). Note that the mathematical manipulations used above are equivalent merely to equating the two pis for the model and for the full-sized ship.

Now if the discussion is not limited to similarly loaded ships, then the quantity  $l^2/S$  will have different values for different ships, and this quantity must be considered. In this case:

$$P/v^2r=f(IV/v, l^2/S)$$

and it will be necessary to plot  $P/v^2r$  versus  $l/v$  for different values of  $l^2/S$  as a parameter.

As a second example, let an expression for the resisting force  $R$  which air offers to the wing of an airplane be found. Considering geometrically similar wings, important quantities are:

$R$ =resisting force

$l$ =a "characteristic dimension" to indicate relative size

$V$ =speed= $L/T$

$r$ =density of air= $M/L^3$

$m$ =viscosity of the air= $M/LT$

There are five important quantities and three dimensions; hence  $5-3=2$  pi's are expected. By the methods previously developed, it may be shown that:

$$P_1=R/rV^2l^2 \quad P_2=lVr/m \quad (\text{Reynolds number})$$

Hence:

$$R/rV^2l^2=f(lVr/m) \quad (5)$$

One could test a model airplane in a wind tunnel and plot  $P_1$  versus  $P_2$ . Also if only a few conditions of operation are desired, one could equate the  $P$ 's as before (where primes refer to the model):

$$R/rV^2l^2=R'/r'V'^2l'^2$$

$$R=rv^2l^2/r'v'^2l'^2R' \quad (6)$$

$$V'=(lrm'/l'r'm')V \quad (7)$$

Equations (6) and (7) illustrate some of the difficulties involved in using models. If one used atmospheric pressure in the wind tunnel thus making  $r$  and  $m$  the same for the model and for the real plane, and if the model were made to a small scale ( $l' \ll l$ ), it follows from (7) that  $V'$  would have to be very great. Also with these conditions,  $R'=R$ ; the force on the model would be as great as that on the actual wing! In attempting to get around these difficulties, three things may be done: (1) use a relatively large scale model, (2) use high pressure (and hence high density) in the test wind tunnel, and (3) make use of the following approximation: Experiment shows that as  $P_2$  in equation (5) is increased, the

(Continued on page 24)



The bus driver charged a lady full fare for her son who was wearing long pants.

At the next corner a small boy wearing short trousers paid only half fare.

At the next corner, a lady boarded the bus and he didn't charge her anything. Why?

You have an evil mind. She had a transfer.

\* \* \*

M. E. Student: "Professor, is water-works spelled as all one word, or is there a hydrant in the middle?"

\* \* \*

Eddey (in EE lab.): "Taylor, grab the end of that wire."

Taylor: "All right, I've got it."

Eddey: "Feel anything?"

Taylor: "No."

Eddey: "Well, then, don't touch the next one to it. It's got 50,000 volts in it."

\* \* \*

"Here's one that Luther Burbank didn't try," said the little coed as she crossed her legs.

\* \* \*

#### Definitions

Chlorine—a dancer in a nite club.

Carbon—a storage place for street cars.

Barium—what you do to dead people.

Boron—a person of low mentality.

Mole—subterranean fur-bearing animal.

Catalyst—a western ranch owner.

Centimeter—a hundred-legged worm-like animal.

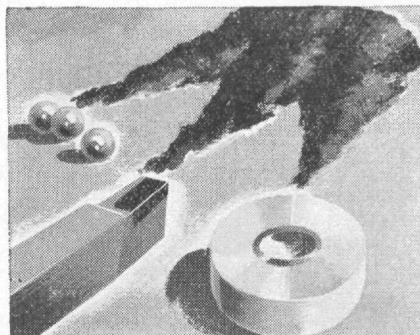
\* \* \*

Prof. (in "culture" class): "Can you tell me anything about the great poets of the 17th century?"

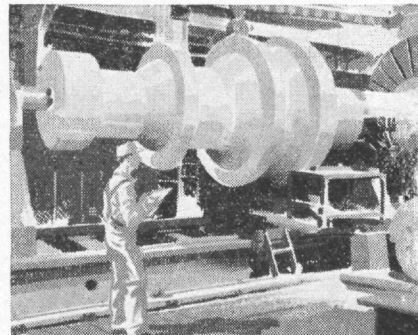
Engineer: "Oh, they're all dead, sir."

# AMAZING FACTS

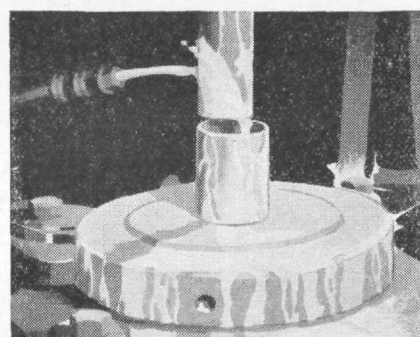
## about the hardest metal made by man



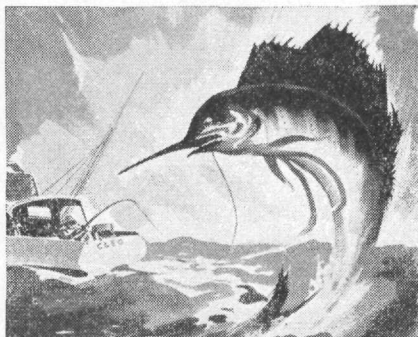
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**AMAZING FACT NO. 2**—The hardest metal made by man works at speeds once thought impossibly high! It has what it takes to machine today's super-tough alloys to tolerances never before possible in mass production. It commonly doubles, *even triples*, the output of machines and men.



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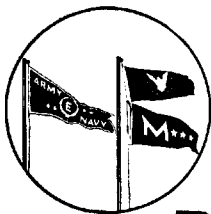
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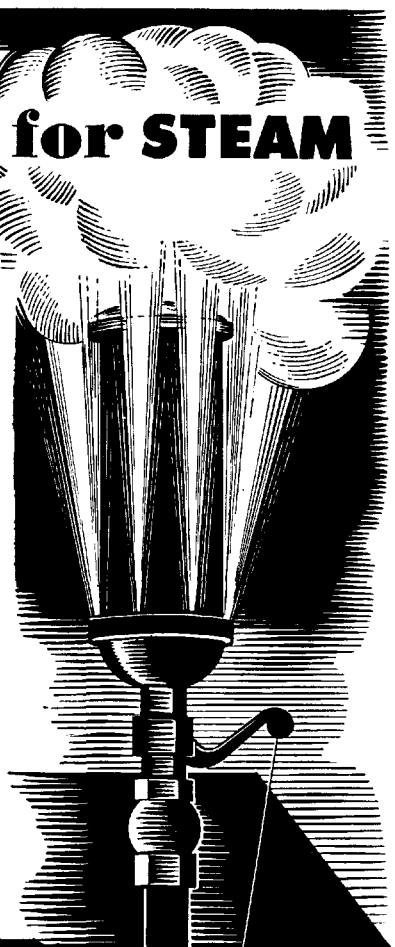


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## BUCKINGHAM PI THEOREM

(Continued from page 22)

function  $f$  tends asymptotically towards a constant value. Hence if  $P_2$  is increased so that the asymptotic region is reached, equation (5) may be written approximately for this region as

$$R = \text{constant} \times r^{1/2} V^2$$

As a final example, consider the problem faced by General Electric Co. engineers in connection with the binary-vapor power plant. It was desired to design a centrifugal pump to pump mercury. The design of centrifugal pumps for water is well developed; and so as a first step in the design, a standard water pump is to be chosen which will give the desired performance with mercury. The specifications which must be met by the mercury pump are a speed of 1750 r.p.m. and a capacity of 2,600,000 lbs/hr of mercury delivered against an exhaust pressure of 500 p.s.i. It is desired to find the operating speed, exhaust pressure, and capacity of the water pump.

Considering geometrically similar pumps, important quantities are:

$l$  = a "characteristic dimension"

$Q$  = quantity of fluid pumped per unit time

$P$  = pressure head to be pumped against

A male nurse in a mental hospital noticed a patient with his ear close to the wall, listening intently. The patient held up a finger as a warning for him to be very quiet; then beckoned him over and said: "You listen here."

The nurse put his ear to the wall and listened for some time, then turned to the patient and said: "I can't hear anything."

"No," said the patient, "and it's been like that all day."

$S$  = speed at which pump is run

$r$  = density of the fluid

$m$  = viscosity of the fluid

Three pi's are anticipated and these are

$$P_1 = Sl^3r/m$$

$$P_2 = l^2Pr/m^2$$

$$P_3 = lQ/m$$

Then letting the subscript  $w$  refer to quantities connected with the water pump, and the subscript  $o$ , to quantities connected with the mercury pump, we have equating the pi's:

$$S_o = S_w (m_o l_w^2 r_w / m_w l_o^2 r_o)$$

$$P_o = (r_w l_w^2 m_o^2 / r_o l_o^2 m_w^2) P_w$$

$$Q_o = (l_w m_o / l_o m_w) Q_w$$

(Continued on page 32)

# The Mt. Vernon Bridge Company

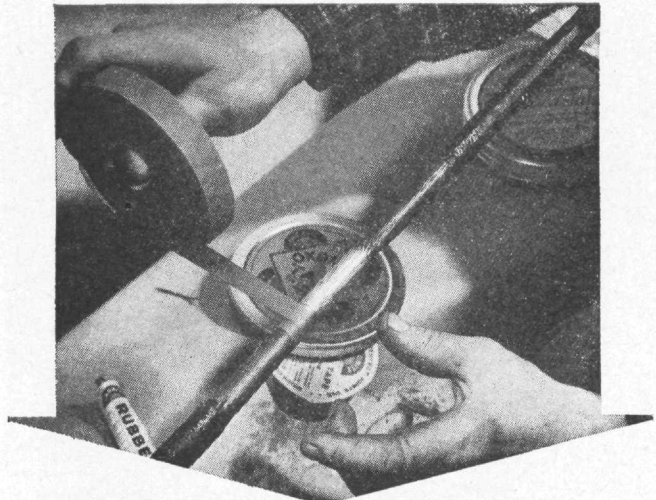
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## BUCKINGHAM PI THEOREM

(Continued from page 24)

Substituting the given values for  $S_o$ ,  $P_o$  and  $Q_o$  in the above equations, letting  $l_o=l_w$ , and making use of the fact that at 20°C:

Water	Mercury
Density=1 gram/cm <sup>3</sup>	Density=13.6
Viscosity=0.0101	grams/cm <sup>3</sup>
grams/cm sec	Viscosity=0.0159
	grams/cm sec

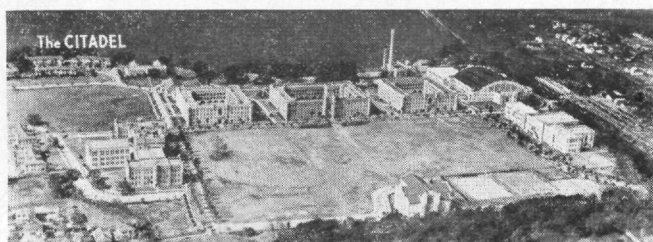
we have:

$$S_o = \frac{.0101}{.0159} (13.6) (1750) = 15,100 \text{ r.p.m.}$$

$$P_o = 13.6 \left( \frac{.0101}{.0159} \right)^2 500 = 2,750 \text{ p.s.i.}$$

$$Q_o = \left( \frac{.0101}{.0159} \right) \frac{2,600,000}{198,000} = 1,265,000 \text{ lbs/hr} = 198,000 \text{ gal/hr}$$

This paper has given some examples of the use of the Buckingham pi theorem. It is hoped that the value of this theorem to engineers has been made apparent.



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And then there was the student who transferred from engineering to arts college and raised the I.Q. of both.

\_\_\_\_\_  
This business of getting up jokes  
Has got me a little bit daunted.  
The ones you want, I can't print  
And the ones I print aren't wanted.

\_\_\_\_\_  
A Frosh came to class the other day with a very black tongue. We questioned him and this was his reply: "I dropped a bottle of whiskey on a freshly-tarred road."

\_\_\_\_\_  
Then there's the one about the veterinarian who tried to breed a mule and a cow. The doc said that he wanted a milk drink with a kick to it.

\_\_\_\_\_  
Q: "Do you know why the little bee buzzes?"  
R: "You'd buzz too, if somebody stole your honey and nectar."

\_\_\_\_\_  
And then there was the rook engineer who let his roommate fix him up with a blind date with with Allis-Chalmers.

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